melt fraction will be more gradual, reflecting the gradual increase of water solubility in olivine and orthopyroxene.

Our results therefore support the concept that the low-velocity zone may be related to partial melting (*1*, *2*, *6*). However, even in the absence of melting, the partitioning of water between olivine and orthopyroxene would strongly depend on depth. The high water solubilities in aluminous orthopyroxene at low pressure and temperature will effectively "dry out" olivine, and this may also contribute to a stiffening of the lithosphere. In any case, however, our results imply that the existence of an asthenosphere—and therefore of plate tectonics as we know it—is possible only in a planet with a water-bearing mantle.

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A Semi-Empirical Approach to Projecting Future Sea-Level Rise

Stefan Rahmstorf

A semi-empirical relation is presented that connects global sea-level rise to global mean surface temperature. It is proposed that, for time scales relevant to anthropogenic warming, the rate of sea-level rise is roughly proportional to the magnitude of warming above the temperatures of the pre–Industrial Age. This holds to good approximation for temperature and sea-level changes during the 20th century, with a proportionality constant of 3.4 millimeters/year per °C. When applied to future warming scenarios of the Intergovernmental Panel on Climate Change, this relationship results in a projected sea-level rise in 2100 of 0.5 to 1.4 meters above the 1990 level.

Understanding global sea-level changes is a difficult physical problem, because complex mechanisms with different time scales play a role (*1*), including thermal expansion of water due to the uptake and penetration of heat into the oceans, input of water into the ocean from glaciers and ice sheets, and changed water storage on land. Ice sheets have the largest potential effect, because their complete melting would result in a global sea-level rise of about 70 m. Yet their dynamics are poorly understood, and the key processes that control the response of ice flow to a warming climate are not included in current ice sheet models [for example, meltwater lubrication of the ice sheet bed (*2*) or increased ice stream flow after the removal of buttressing ice shelves (*3*)]. Large uncertainties exist even in the projection of thermal expansion, and estimates of the total volume of ice in mountain glaciers and ice caps that are remote from the continental ice sheets are uncertain by a factor of two (*4*). Finally, there are as yet no published physically based projections of ice loss from glaciers and ice caps fringing Greenland and Antarctica.

For this reason, our capability for calculating future sea-level changes in response to a given surface warming scenario with present physics-based models is very limited, and models are not able to fully reproduce the sea-level rise of recent decades. Rates of sea-level rise calculated with climate and ice sheet models are generally lower than observed rates. Since 1990, observed sea level has followed the uppermost uncertainty limit of the Intergovernmental Panel on Climate Change (IPCC) Third Assessment Report (TAR), which was constructed by assuming the highest emission scenario combined with the highest climate sensitivity and adding an ad hoc amount of sea-level rise for "ice sheet uncertainty" (*1*).

While process-based physical models of sea-level rise are not yet mature, semi-empirical models can provide a pragmatic alternative to estimate the sea-level response. This is also the approach taken for predicting tides along coasts (for example, the well-known tide tables), where the driver (tidal forces) is known, but the calculation of the sea-level response from first principles is so complex that semi-empirical relationships perform better. Likewise, with current and future sea-level rise, the driver is known [global warming (*1*)], but the computation of the link between the driver and the response from first principles remains elusive. Here, we will explore a semi-empirical method for estimating sea-level rise.

As a driver, we will use the global average near-surface air temperature, which is the standard diagnostic used to describe global warming. Figure 1 shows a schematic response to a step-function increase in temperature, after climate and sea level parameters were at equilibrium. We expect sea level to rise as the ocean takes up heat and ice starts to melt, until (asymptotically) a new equilibrium sea level is reached. Paleoclimatic data suggest that changes in the final equilibrium level may be very large: Sea level at the Last Glacial Maximum, about 20,000 years ago, was 120 m lower than the current level, whereas global mean temperature was 4° to 7°C lower (*5*, *6*). Three million years ago, during the Pliocene, the average climate was about 2° to 3°C warmer and sea level was

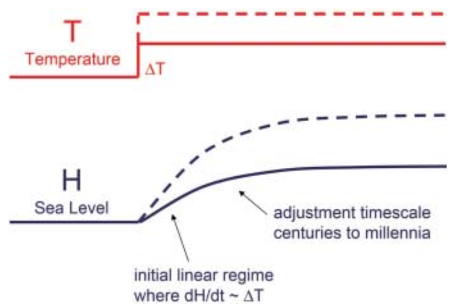

Fig. 1. Schematic of the response of sea level to a temperature change. The solid line and the dashed line indicate two examples with different amplitude of temperature change.

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25 to 35 m higher (*7*) than today's values. These data suggest changes in sea level on the order of 10 to 30 m per °C.

The initial rate of rise is expected to be proportional to the temperature increase

$$dH/dt = a\ (T - T_0) \quad (1)$$

where H is the global mean sea level, t is time, a is the proportionality constant, T is the global mean temperature, and T_0 is the previous equilibrium temperature value. The equilibration time scale is expected to be on the order of millennia. Even if the exact shape of the time evolution $H(t)$ is not known, we can approximate it by assuming a linear increase in the early phase; the long time scales of the relevant processes give us hope that this linear approximation may be valid for a few centuries. As long as this approximation holds, the sea-level rise above the previous equilibrium state can be computed as

$$H(t) = a\int_{t0}^{t}(T(t') - T_0)\ dt' \quad (2)$$

where t' is the time variable.

We test this relationship with observed data sets of global sea level (*8*) and temperature [combined land and ocean temperatures obtained from NASA (*9*)] for the period 1880–2001, which is the time of overlap for both series. A highly significant correlation of global temperature and the rate of sea-level rise is found ($r = 0.88$, $P = 1.6 \times 10^{-8}$) (Fig. 2) with a slope of a = 3.4 mm/year per °C. If we divide the magnitude of equilibrium sea-level changes that are suggested by paleoclimatic data (*5*–*7*) by this rate of rise, we obtain a time scale of 3000 to 9000 years, which supports the long equilibration time scale of sea-level changes.

The baseline temperature T_0, at which sea-level rise is zero, is 0.5°C below the mean temperature of the period 1951–1980. This result is consistent with proxy estimates of temperatures in the centuries preceding the modern warming (*10*), confirming that temperature and sea level were not far from equilibrium before this modern warming began. This is consistent with the time scale estimated above and the relatively stable climate of the Holocene (the past 10,000 years).

In Fig. 3, we compare the time evolution of global mean temperature, converted to a "hindcast" rate of sea-level rise according to Eq. 1, with the observed rate of sea-level rise. This comparison shows a close correspondence of the two rates over the 20th century. Like global temperature evolution, the rate of sea-level rise increases in two major phases: before 1940 and again after about 1980. It is this figure that most clearly demonstrates the validity of Eq. 1. Accordingly, the sea level that was computed by integrating global temperature with the use of Eq. 2 is in excellent agreement with the observed sea level (Fig. 3), with differences always well below 1 cm.

We can explore the consequences of this semi-empirical relationship for future sea levels (Fig. 4), using the range of 21st century temperature scenarios of the IPCC (*1*) as input into Eq. 2. These scenarios, which span a range of temperature increase from 1.4° to 5.8°C between 1990 and 2100, lead to a best estimate of sea-level rise of 55 to 125 cm over this period. By including the statistical error of the fit shown in Fig. 2 (one SD), the range is extended from 50 to 140 cm. These numbers are significantly higher than the model-based estimates of the IPCC for the same set of temperature scenarios, which gave a range from 21 to 70 cm (or from 9 to 88 cm, if the ad hoc term for ice sheet uncertainty is included). These semi-empirical scenarios smoothly join with the observed trend in 1990 and are in good agreement with it during the period of overlap.

We checked that this analysis is robust within a wide range of embedding periods (i.e., smoothing) of the observational time series. The slope found in Fig. 2 varies between 3.2 and 3.5 mm/year per °C for any embedding period between 2 and 17 years, causing only minor variations in the projected sea level. For short embedding periods (around 5 years), the rate of sea-level rise (Fig. 3, top) closely resembles that shown in (*8*) with large short-term fluctuations. For embedding dimensions longer than 17 years, the slope starts to decline, because the acceleration of sea-level rise since 1980 (Fig. 3) is then progressively lost by excessive smoothing. For very long embedding periods (30 years), the rate of sea-level rise becomes rather flat such as that shown in (*11*).

The linear approximation (Eq. 1) is only a simplistic first-order approximation to a number of complex processes with different time scales. The statistical error included in Fig. 4 does not include any systematic error that arises if the linear relationship breaks down during the forecast period. We can test for this systematic error using climate models, if only for the thermal expansion component of sea-level rise that these models capture. For this test, we used the CLIMBER-3α climate model (*12*), which uses a simplified atmosphere model coupled to a three-dimensional general circulation ocean model with free surface (i.e., that vertically adjusts). We used a model experiment initialized from an equilibrium state of the coupled system in the year 1750 and, with historic radiative forcing, forced changes until the year 2000. After 2000, the model was forced with the IPCC A1FI scenario. The global mean temperature increases by 0.8°C in the 20th century and by 5.0°C from 1990 to 2100 in this experiment.

Temperature and sea-level rise data from this model for the time period 1880–2000 were treated like the observational data in the analysis presented above, and graphs corresponding to Figs. 2 and 3 look similar to those derived from the observational data (figs. S1 and S2). The slope found is only 1.6 mm/year per °C (i.e., half of the observed slope) because only the thermal expansion component is modeled. Using the semi-empirical relation as fitted to the period 1880–2000, we predicted the sea level for the 21st century (fig. S3). Up to the year 2075, this predicted sea level remains within 5 cm of the actual (modeled) sea level. By the year 2100, the predicted level is 51 cm whereas the actual (modeled) level is 39 cm above that of 1990 (i.e., the semi-empirical formula overpredicts sea level by 12 cm).

For the continental ice component of sea-level rise, we do not have good models to test how the

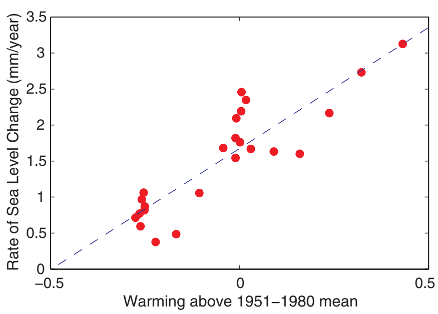

Fig. 2. Correlation of temperature and the rate of sea-level rise for the period 1881–2001. The dashed line indicates the linear fit. Both temperature and sea-level curves were smoothed by computing nonlinear trend lines, with an embedding period of 15 years (*14*). The rate of sea-level change is the time derivative of this smoothed sea-level curve, which is shown in Fig. 3. Data were binned in 5-year averages to illustrate this correlation.

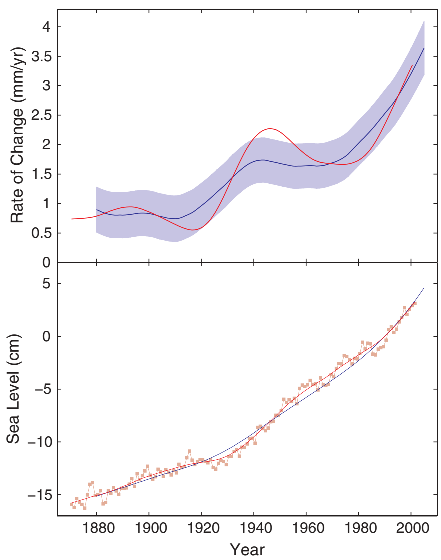

Fig. 3. (**Top**) Rate of sea-level rise obtained from tide gauge observations (red line, smoothed as described in the Fig. 2 legend) and computed from global mean temperature from Eq. 1 (dark blue line). The light blue band indicates the statistical error (one SD) of the simple linear prediction (*15*). (**Bottom**) Sea level relative to 1990 obtained from observations (red line, smoothed as described in the Fig. 2 legend) and computed from global mean temperature from Eq. 2 (blue line). The red squares mark the unsmoothed, annual sea-level data.

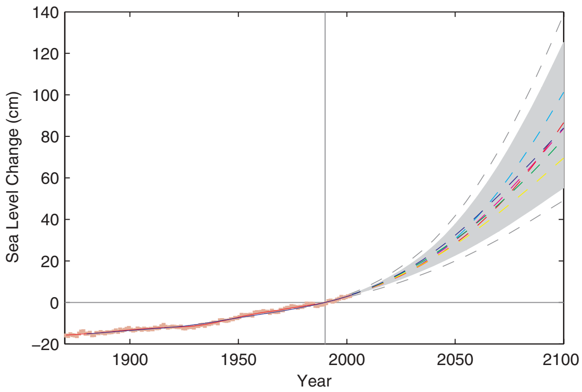

Fig. 4. Past sea level and sea-level projections from 1990 to 2100 based on global mean temperature projections of the IPCC TAR. The gray uncertainty range spans the range of temperature rise of 1.4° to 5.8° C, having been combined with the best statistical fit shown in Fig. 2. The dashed gray lines show the added uncertainty due to the statistical error of the fit of Fig. 2. Colored dashed lines are the individual scenarios as shown in (*1*); the light blue line is the A1FI scenario, and the yellow line is the B1 scenario.

linear approximation performs, although the approximation is frequently used by glaciologists ("degree-days scheme"). Given the dynamical response of ice sheets observed in recent decades and their growing contribution to overall sea-level rise, this approximation may not be robust. The ice sheets may respond more strongly to temperature in the 21st century than would be suggested by a linear fit to the 20th century data, if time-lagged positive feedbacks come into play (for example, bed lubrication, loss of buttressing ice shelves, and ocean warming at the grounding line of ice streams). On the other hand, many small mountain glaciers may disappear within this century and cease to contribute to sea-level rise. It is therefore difficult to say whether the linear assumption overall leads to an over- or underestimation of future sea level. Occam's razor suggests that it is prudent to accept the linear assumption as reasonable, although it should be kept in mind that a large uncertainty exists, which is not fully captured in the range shown in Fig. 4.

Regarding the lowest plausible limit to sea-level rise, a possible assumption may be that the rate shown in Fig. 3 stops increasing within a few years (although it is difficult to see a physical reason for this) and settles at a constant value of 3.5 mm/year. This implies a sea-level rise of 38 cm from 1990 to 2100. Any lower value would require that the rate of sea-level rise drops despite rising temperature, reversing the relationship found in Fig. 2.

Although a full physical understanding of sea-level rise is lacking, the uncertainty in future sea-level rise is probably larger than previously estimated. A rise of over 1 m by 2100 for strong warming scenarios cannot be ruled out, because all that such a rise would require is that the linear relation of the rate of sea-level rise and temperature, which was found to be valid in the 20th century, remains valid in the 21st century. On the other hand, very low sea-level rise values as reported in the IPCC TAR now appear rather implausible in the light of the observational data.

The possibility of a faster sea-level rise needs to be considered when planning adaptation measures, such as coastal defenses, or mitigation measures designed to keep future sea-level rise within certain limits [for example, the 1-m long-term limit proposed by the German Advisory Council on Global Change (*13*)].

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Nonequilibrium Mechanics of Active Cytoskeletal Networks

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Cells both actively generate and sensitively react to forces through their mechanical framework, the cytoskeleton, which is a nonequilibrium composite material including polymers and motor proteins. We measured the dynamics and mechanical properties of a simple three-component model system consisting of myosin II, actin filaments, and cross-linkers. In this system, stresses arising from motor activity controlled the cytoskeletal network mechanics, increasing stiffness by a factor of nearly 100 and qualitatively changing the viscoelastic response of the network in an adenosine triphosphate–dependent manner. We present a quantitative theoretical model connecting the large-scale properties of this active gel to molecular force generation.

Mechanics directly control many functions of cells, including the generation of forces, motion, and the sensing of external forces (*1*). The cytoskeleton is a network of semiflexible linear protein polymers (actin filaments, microtubules, and intermediate filaments) that is responsible for most of the mechanical functions of cells. It differs from common polymer materials in both the complexity of composition and the fact that the system is not at thermodynamic equilibrium. Chemical nonequilibrium drives mechanoenzymes (motor proteins) that are the force generators in cells. The cytoskeleton is thus an active material that can adapt its mechanics and perform mechanical tasks such as cell locomotion or cell division.

Here, we show how nonequilibrium motor activity controls the mechanical properties of a simple three-component in vitro model cytoskeletal network. The nonequilibrium origin of this active control mechanism can be seen directly in the violation of a fundamental theorem of statistical physics, the fluctuation-dissipation (FD) theorem, which links thermal fluctuations of systems to their response to external forces. The FD theorem is a generalization of Einstein's description of Brownian motion (*2*). Although it is valid only in equilibrium, its possible extension to out-of-equilibrium systems such as granular materials and living cells has been debated (*3*–*5*). Prior studies in cells have suggested violations of the FD theorem (*3*), but this has not been directly observed. We show that an in vitro model system consisting of a cross-linked

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Comment on "A Semi-Empirical Approach to Projecting Future Sea-Level Rise"

Simon Holgate,[1]* Svetlana Jevrejeva,[1] Philip Woodworth,[1] Simon Brewer[2]

Rahmstorf (Reports, 19 January 2007, p. 368) presented an approach for predicting sea-level rise based on a proposed linear relationship between global mean surface temperature and the rate of global mean sea-level change. We find no such linear relationship. Although we agree that there is considerable uncertainty in the prediction of future sea-level rise, this approach does not meaningfully contribute to quantifying that uncertainty.

Rahmstorf (*1*) proposed a relationship between global mean surface temperatures (*2*, *3*) and the rate of global mean sea-level change (*4*). The approach assumes that "the rate of sea-level rise is roughly proportional to the magnitude of warming above the temperatures of the pre–Industrial Age" (*1*). On this basis, sea level is predicted to rise 0.5 to 1.4 m above the 1990 level by 2100. These estimates are considerably higher than those published in the Third Assessment Report of the Intergovernmental Panel on Climate Change (*5*) and therefore require closer inspection.

The calculation of the linear relationship between temperature and the rate of sea-level change (*1*) did not explore whether the calculated proportionality constant of 3.4 mm/year per °C applies to the time scales of most relevance to anthropogenic warming (i.e., decades to centuries). Figure 1A replicates figure 2 in (*1*). As in (*1*), both the temperature and sea-level time series are smoothed with the Monte Carlo singular spectrum analysis method (MC-SSA) (*6*) to remove energy with periods of less than 15 years. However, we split the data into four epochs that approximately relate to the four dominant periods of the temperature record (Fig. 1B), and we did not apply the 5-year binning procedure as in (*1*), because that further reduces the degrees of freedom. Figure 1A clearly demonstrates that no linear relationship exists on a 50-year time scale, which is 50% of the 100-year period for which predictions were made in (*1*).

We note that using the model $dH/dt = a(T-T_0)$ (where a is the proportionality constant, T is the global mean temperature, and T_0 is the previous equilibrium temperature value), with the quoted values of $a = 3.4$ mm/year per °C and $T_0 = -0.5$°C (*1*), gives $dH/dt = 1.7$ mm/year with zero (average) change in temperature (i.e., with $T = 0$). This shows that the mean rate obtained from this model over the past century agrees well with other estimates of sea-level rise over the past 100 years [e.g., (*4*, *7*)]. However, the issue is whether this model can provide information at shorter periods than the century scale and be used to predict global sea levels some decades into the future.

A reasonable test of the strength of a model is its ability to predict observations that are not already included in its formulation. To illustrate the nonlinearity of the temperature/sea-level change relationship, we calculated linear coefficients for the first half of the observational record and then proceeded to predict the remaining observations. We also used the second half of the data set to hindcast sea levels during the earlier part of the record. To make this testing sensitive to changes on time scales of decades, which are of most interest for prediction, we detrended both the smoothed surface temperatures and the smoothed sea levels for the first and second halves of the data before calculating the annual rates of sea-level change (detrending improves the results but does not change their character). We then calcu-

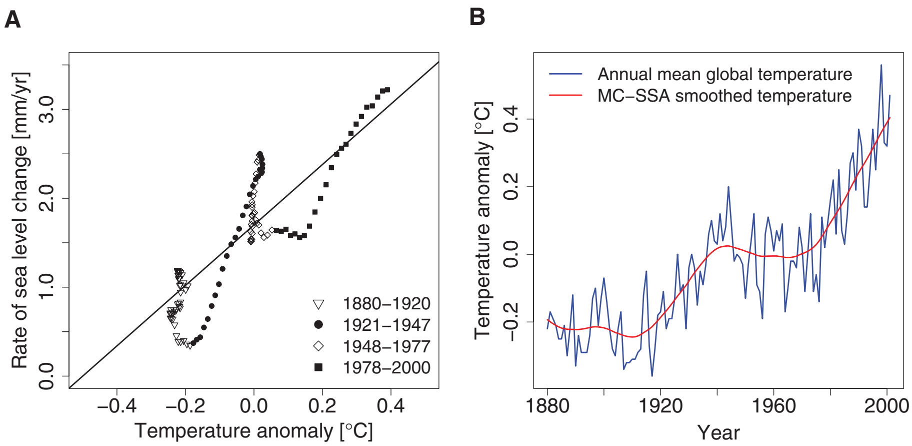

Fig. 1. (**A**) The relationship of the rate of global mean sea-level rise (*4*) to global mean surface temperature (*2*, *3*) with the data divided into four epochs, each showing a different relationship between the variables. This figure is similar to figure 2 in (*1*) but without the binning into 5-year averages so as to better illustrate the data clustering. (**B**) The global mean surface temperature record (*2*, *3*), annual data and data smoothed using the MC-SSA method (*6*). The four epochs described in (A) relate to the four sections of the temperature record that can be clearly seen.

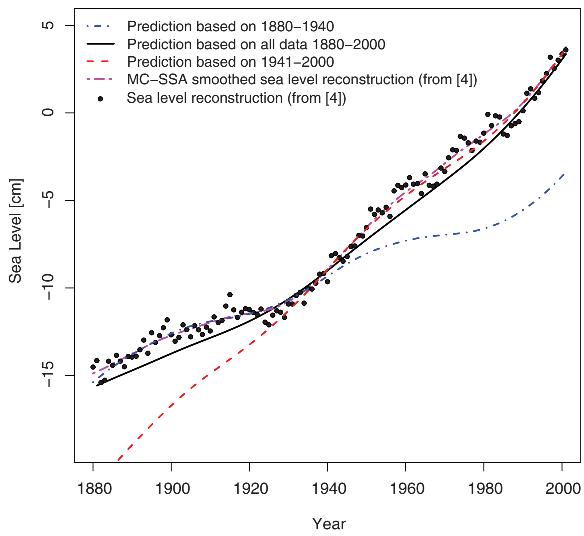

Fig. 2. Hindcasts of the global mean sea level based on linear rates calculated from the full data set as in (*1*) and based on rates calculated from the first and second halves of the reconstructed sea-level record (*4*). The mean rate of sea-level rise is 0.86 mm/year based on the first half of the record and 1.98 mm/year based on the second half of the record. The mean rate for the 1887 to 1994 period based on the sea-level reconstruction (*4*) is 1.49 mm/year.

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lated the linear regression coefficients for the two halves of the data (a_1, a_2) along with the equivalent values of T_0 (i.e., T_{01}, T_{02}). Finally, to obtain the full dH/dt, we added back the linear trend, which had been subtracted from the sea levels. From the above regression, we obtain a_1 = 8.26 mm/year per °C and T_{01} = −0.12°C for the first half of the data and a_2 = 6.60 mm/year per °C and T_{02} = −0.32°C for the second half, compared with a = 3.4 mm/year per °C and T_0 = −0.5°C from fitting the whole data set. The root mean square error is 0.21 mm over the first half of the record to which the data are fitted and 0.35 mm over the second half of the record when the data are fitted to that. This is in comparison with 0.62 mm for the model when fitted to all the data, illustrating that we do indeed obtain a better fit to the data included in the model over shorter time periods. The results of this analysis are shown in Fig. 2, which shows that at the 50- to 100-year time scale, the linear relationship has little skill in predicting the observations not included in the original model formulation. Using the coefficients obtained from the first half of the data, a trend in sea level of 0.86 mm/year is predicted for the entire 122-year period, whereas using the second half of the data, a trend of 1.98 mm/year is calculated. These compare with a trend of 1.49 mm/year for the sea-level reconstruction (*4*) to the time period over which the model is formulated.

In conclusion, although we agree that there is considerable uncertainty in future projections of sea-level change and that model predictions currently appear to underestimate observations, we do not agree that simplistic projections of the nature presented in (*1*) substantially contribute to our understanding of the uncertainties in the nonlinear relationships of the climate system.

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Comment on "A Semi-Empirical Approach to Projecting Future Sea-Level Rise"

Torben Schmith,[1]* Søren Johansen,[2] Peter Thejll[3]

Rahmstorf (Reports, 19 January 2007, p. 368) used the observed relation between rates of change of global surface temperature and sea level to predict future sea-level rise. We revisit the application of the statistical methods used and show that estimation of the regression coefficient is not robust. Methods commonly used within econometrics may be more appropriate for the problem of projected sea-level rise.

Rahmstorf (*1*) convincingly argued for the use of semi-empirical models for estimating sea-level response to future warming of the climate system. He hypothesized that the rate of global sea-level change is proportional to the global surface temperature departure from its equilibrium value. This hypothesis was statistically tested on observational data and a correlation coefficient of 0.88 was reported along with an associated P value of 1.6×10^{-8} and a regression slope of 3.4 mm/year per °C. We argue that this statistical analysis is based on an inappropriate application of statistics, in that the trend in both series is evident, thus violating basic assumptions of the statistical methods used. This could give misleading conclusions about inference (*2*) due to a spurious correlation (*3*) and, as such, casts doubt on the projected range of future sea-level rise.

To illustrate this problem, we reperformed the analysis of Rahmstorf (*1*), with methodological details as follows: As in (*1*), we used annually averaged global mean temperature and sea level. Nonlinear trends of both series were determined as the first reconstructed component in a singular spectrum analysis (SSA) with an embedding dimension of 15 years. Before the SSA analysis, both series were extended forward and backward by linear extrapolation based on the nearest 15 years of data. The nonlinear trend of mean sea level was subsequently differentiated by calculating, at each point, the slope of a ±5-year least squares fit to obtain a "rate of sea level change" series. The correlation coefficient ρ between two time series x_t and y_t with deterministic time trends is defined as

$$\rho = \frac{E\{[x_t - E(x_t)][y_t - E(y_t)]\}}{\sqrt{E\{[x_t - E(x_t)]^2\}E\{[y_t - E(y_t)]^2\}}}$$

where E denotes expectation value. The correlation coefficient thus measures the degree to which there are coincident departures of the two time series from their respective expectation values. When estimating the correlation coefficient between filtered versions of temperature and rate of sea-level change, Rahmstorf (*1*) substituted the expectation values by the sample average. This assumes stationarity of the series (*4*), which is obviously violated by the two series. As an illustration, when redoing the analysis, we approximated the expectation values of the two series by the more realistic choice of a linear trend. The estimated correlation coefficient then drops to 0.68. In addition, the corresponding regression coefficient increases from 3.3 mm/year per °C to 5.8 mm/year per °C. This nonrobust result underscores that the issue of correct statistical modeling is not an academic one and raises questions about the model put forward in (*1*).

Next, there is the point of establishing the significance of the correlation coefficient found, that is, "how likely is it to get the result by pure chance?" Rahmstorf appears to have estimated the P value of the correlation coefficient (1.6×10^{-8}) using a Student's t distribution assuming 24 degrees of freedom. The number of degrees of freedom apparently comes from assuming that the 24 bins of 5-year length are statistically independent. However, both data series were low-pass filtered in (*1*) "by computing nonlinear trend lines, with an embedding period of 15 years." Because of the autocorrelation introduced by the averaging procedure, neighboring 5-year bins can not be assumed to be statistically independent. A better approximation would be to set the number of degrees of freedom equal to the effective sample size calculated as 120/15 = 8 (*4*). Using our value of 0.68 for the correlation coefficient, we get a corresponding P value of 0.97.

Finally, Rahmstorf used a t test for making inferences about the correlation coefficient. This is based on the assumption of independent and identically distributed (i.i.d.) data, yet the data analyzed in (*1*) are, due to the trend, not i.i.d. This reservation also applies to the estimated confidence interval and, therefore, the range of projected sea-level changes by the year 2100. A thorough analysis of the problem would include the application of difference stationary time series analysis methods (*5*), for which there is a rich tradition in the field of econometrics. Such analysis may also be helpful to other problems in climate science.

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Response to Comments on "A Semi-Empirical Approach to Projecting Future Sea-Level Rise"

Stefan Rahmstorf

Additional analysis performed in response to Holgate *et al.* and Schmith *et al.* shows that the semi-empirical method for projecting future sea-level rise passes the test of predicting one half of the data set based on the other half. It further shows that the conclusions are robust with respect to choices of data binning, smoothing, and detrending.

The technical comments by Holgate *et al.* (*1*) and Schmith *et al.* (*2*) provide a welcome opportunity to present further analysis of the link between sea-level rise and global warming, and to make the computer code used in the analysis available for use by other researchers (see Supporting Online Material).

Holgate *et al.* raise two issues. The first, shown in Fig. 1 in (*1*), concerns what they call a "clustering" of data in the scatter plot of temperature versus rate of sea-level rise. However, this clustering is an artifact of the authors' plotting annual data points based on a 15-year smoothed sea-level record, resulting in data points that are not independent but highly autocorrelated. This is the reason I binned the data points in my scatter plot [figure 2 in (*3*)]. This does not "further reduce the degrees of freedom," as Holgate *et al.* claim, but rather reflects the fact that there simply are not more degrees of freedom in these data after the smoothing. In fact, as the comment by Schmith *et al.* (*2*) correctly observes, it would have been more consistent to use 15-year bins. Using 15-year bins, $r = 0.9$ and $P = 0.002$ including the trend, or $r = 0.7$ and $P = 0.04$ for a detrended version of the analysis (see below). Thus, the correlation is still significant at the 99% level with trend and at the 95% level even without trend. Note that the binning affects only the look of the graph, not the statistical fit (i.e., slope and base temperature), and the particular smoothing procedure used has only a minor impact. The future sea level projections presented in (*3*) are robust to changes in these technicalities of the analysis.

The second issue that Holgate *et al.* raise is whether the semi-empirical formula proposed in (*3*) passes a simple test of predicting one half of the data set based on the other half of the data set. That this is indeed the case is demonstrated in Figs. 1 and 2. Figure 1 shows the predicted rate of sea-level rise, and of sea level itself, exactly as in figure 3 in (*3*), but using only the first half of the data set (1880 to 1940) for deriving the statistical fit. The slope found in this case is 0.42 mm/year per °C, and the base temperature is –0.42°C (relative to the period 1951 to 1980). The result shows that sea level for the period 1940 to 2000 is predicted well (to within 2 cm of observed sea level) by the semi-empirical formula, based only on the sea-level data before 1940. Figure 2 shows the same for a hindcast of sea level for the period 1880 to 1940, based only on the data after 1940 (in this case, sea level is integrated backward from the present). In each case, the error margins (dashed lines) are small enough to give useful predictions despite using only 60 years of data, and the observed sea level is well within those error margins of the method. These error margins are computed the same way as shown by the dashed gray lines in figure 4 in (*3*). The semi-empirical method thus passes this simple test very well, and its validity is thereby confirmed. The algorithms used here are the same as in (*3*). The fact that Holgate *et al.* show different results in their figure 2 is due to their using a different method, which involves detrending each half of the data separately (and likely some other differences). Comparing the graphs shows that the performance of their method is not as good as that of the method used in (*3*). The acceleration in sea-level rise between the first period (1880 to 1940) and the second period (1940 to 2000) due to global warming is captured by my semi-empirical model but not by the alternative approach proposed by Holgate *et al.*

The comment by Schmith *et al.* (*2*) further raises the issue of the trend of the series being included in the correlation. Whether an analysis with trend or after removal of a linear (or higher order) trend is more useful depends on what one

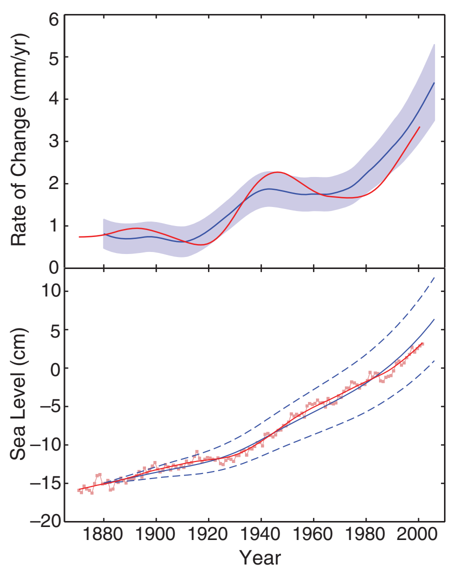

Fig. 1. (**Top**) Observed rate of sea-level rise (red) and that forecast using the simple empirical model (blue), trained using data for the period 1880 to 1940. (**Bottom**) Observed sea level (red) and that predicted using the empirical model (blue), by integrating the blue curve from the top panel forward in time. Dashed lines show the error estimate for the prediction, as in (*3*).

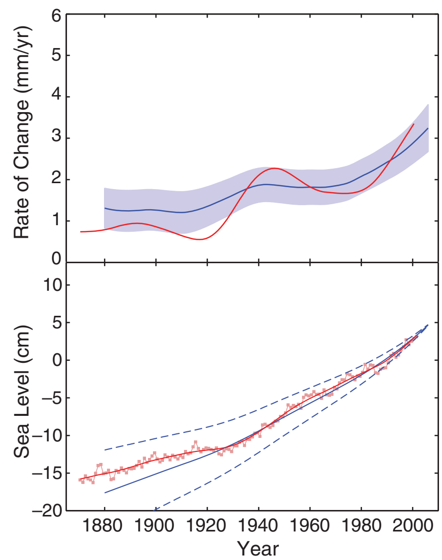

Fig. 2. (**Top**) Observed rate of sea-level rise (red) and that forecast using the simple empirical model (blue), trained using data for the period 1940 to 2000. (**Bottom**) Observed sea level (red) and that predicted using the empirical model (blue), by integrating the blue curve from the top panel backward in time. Dashed lines show the error estimate for the prediction, as in (*3*).

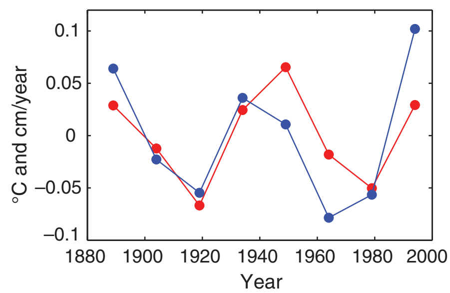

Fig. 3. Fifteen-year averages of the global mean temperature (blue, °C) and rate of sea level rise (red, cm/year), both detrended.

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is interested in. In this case, the common trend of global temperature and the rate of sea-level rise is one of the most interesting aspects of the data. If the rate of sea-level rise had not increased while temperatures warmed, the basic idea behind my analysis would have been falsified right away. Nevertheless, even the detrended series show a strong and significant correlation, with $r = 0.7$. This is evident from Fig. 3, which shows the temperature (blue) and the rate of sea-level rise (red) in their detrended versions using 15-year bins. Using the detrended data for the fit, the agreement with past observed sea level is not quite as good, the sea-level projections for the year 2100 are raised by about one-third (e.g., to 93 cm instead of 69 cm for the B1 scenario), and the statistical error estimate for these projections is increased by up to a factor of three.

Schmith *et al*. also raise the possibility of "nonsense correlations," that is, real correlations that do not have a causal basis. This can of course never be ruled out; data can only falsify but never prove a hypothesis. However, the starting point of my analysis and my paper was not a correlation found in data but rather the physical reasoning that a change in global temperature should to first order be proportional to a change in the rate of sea-level rise. The analysis shows that the data of the past 120 years are indeed consistent with this expectation, and the expected connection is statistically significant. The observational data therefore strongly support the hypothesis I put forward.

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Supporting Online Material

www.sciencemag.org/cgi/content/full/317/5846/1866d/DC1
Computer algorithm and data files

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